

dimensionless parameter;  $\epsilon = R_2 \rho_2 u_2 c_2 / R_1 \rho_1 u_1 c_1$ , dimensionless parameter;  $\kappa = (\lambda_T / \lambda_1)(R_1 / 2b)$ , dimensionless parameter;  $Pe_i = \rho_i u_i c_i R_i / \lambda_i$ , thermal Péclet number. Subscripts: s, surface of separating membrane; i = 1, 2, "hot" and "cold" channels, respectively; l, outlet of gas; T, separating membrane.

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#### DETERMINING THE THERMOPHYSICAL CHARACTERISTICS OF MATERIALS ON A MODEL OF A SEMIINFINITE BODY WITH HEAT SUPPLIED BY MEANS OF A THIN ANNULAR HEATER

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Methods of complex calculation of the thermophysical characteristics of materials without destroying their integrity are proposed, on the basis of a model of a semiinfinite body on heating with a rectangular heat pulse through a specified annular region.

The problem posed here is the complex determination of the thermophysical characteristics of materials (without loss of their integrity) using a model of a semiinfinite (in thermal terms) body with a pulsed heat supply to its surface. A short heat pulse (of rectangular form) acts in a limited annular region, and the excess temperature  $T_i(r, z, \tau)$  is measured at a point coinciding with the center of the annular heat source.

To solve this problem the two-dimensional nonsteady temperature field  $T_i(r, z, \tau)$  in cylindrical coordinates must be determined as a function of the heat-flux density  $q(\tau)$  acting in a finite annular region  $R_1 \leq r \leq R_2$ , where  $R_1$  and  $R_2$  are the radii of the annular heater at the surface of the semiinfinite body when  $z = 0$ ;  $R_2 > R_1$ . In the regions of variation  $r < R_1$  and  $R_2 < r < \infty$  on the surface, there is assumed to be no temperature gradient along the normal to the boundary of the body. The initial temperature distribution is assumed to be constant:  $T_0 = \text{const}$ . The origin of the cylindrical coordinates ( $r = z = 0$ ) is chosen at the center of the annular heater.

The mathematical problem is formulated as a system of three differential heat-conduction equations of the form

$$\frac{\partial^2 T_i(r, z, \tau)}{\partial r^2} + \frac{1}{r} \frac{\partial T_i(r, z, \tau)}{\partial r} + \frac{\partial^2 T_i(r, z, \tau)}{\partial z^2} = \frac{1}{a} \frac{\partial T_i(r, z, \tau)}{\partial \tau} \quad (i = 1, 2, 3). \quad (1)$$

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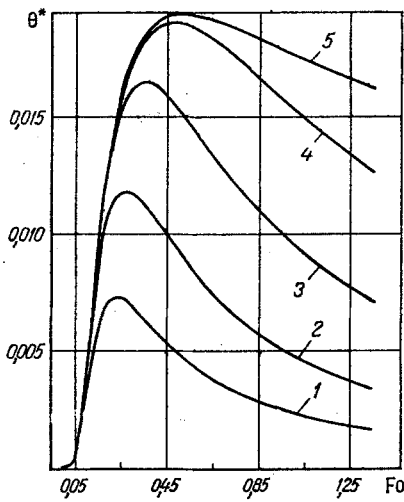


Fig. 1

Fig. 1. Functions  $\Theta^*(Fo, \varphi_0, K_R)$  and  $\Theta_1^*(Fo, \varphi_0)$ :  $Fo_0=0.04$ ; 1)  $K_R = 1.25$ ; 2) 1.5; 3) 2.0; 4) 3.0; 5)  $K_R = \infty$ .

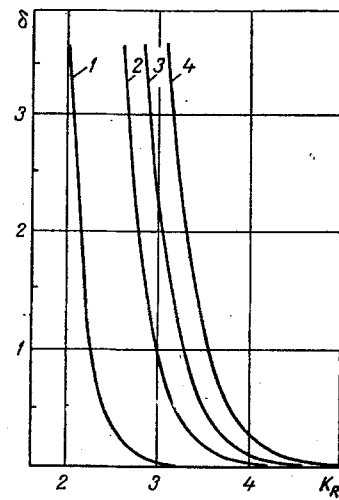


Fig. 2

Fig. 2. Curves of  $\delta, \%$ , as a function of  $K_R$ : 1)  $Fo = 0.25$ ; 2) 0.45; 3) 0.55, 4)  $Fo = 0.65$ .

The temperature fields  $T_i(r, z, \tau)$  correspond to the changes in the variables  $r, z, \tau$  in the following ranges

$$\text{for } T_1(r, z, \tau) \quad 0 \leq r < R_1, \quad z \geq 0, \quad \tau > 0;$$

$$\text{for } T_2(r, z, \tau) \quad R_1 < r < R_2, \quad z \geq 0, \quad \tau > 0;$$

$$\text{for } T_3(r, z, \tau) \quad R_2 < r < \infty, \quad z \geq 0, \quad \tau > 0.$$

The boundary conditions for Eq. (1) are simple to specify, bearing in mind that for the temperature field  $T_1(r, z, \tau)$  at  $r = 0$  symmetry of the temperatures and heat fluxes relative to the axis  $z \geq 0$  is observed at any time  $\tau$ , while at the boundaries of the cylindrical surfaces  $r = R_1$  and  $r = R_2$  the matching conditions are satisfied, i.e., the values of the corresponding temperatures and heat fluxes (over the radius) are equal at any  $z \geq 0, \tau > 0$ .

The solution of the problem for  $\Delta T_i(r, z, \tau) = T_i(r, z, \tau) - T_0$  is obtained by operative methods and takes the form

$$\begin{aligned} \Delta T_1(r, z, \tau) = & \frac{R_1}{\lambda} \frac{1}{\pi^2 i} \int_0^{\sigma+i\infty} \int_{\sigma-i\infty}^{\sigma+i\infty} \times \\ & \times \exp(\sigma\tau) \frac{I_0\left(r\sqrt{p^2 + \frac{s}{a}}\right) K_1\left(R_1\sqrt{p^2 + \frac{s}{a}}\right)}{\sqrt{p^2 + \frac{s}{a}}} \bar{q}(s) \cos pz ds p - \\ & - \frac{R_2}{\lambda} \frac{1}{\pi^2 i} \int_0^{\sigma+i\infty} \int_{\sigma-i\infty}^{\sigma+i\infty} \exp(\sigma\tau) \frac{I_0\left(r\sqrt{p^2 + \frac{s}{a}}\right) K_1\left(R_2\sqrt{p^2 + \frac{s}{a}}\right)}{\sqrt{p^2 + \frac{s}{a}}} \times \bar{q}(s) \cos pz ds p; \\ \Delta T_2(r, z, \tau) = & \frac{1}{b\sqrt{\pi}} \int_0^{\tau} \exp\left[-\frac{z^2}{4a(\tau-\xi)}\right] \frac{q(\xi)}{\sqrt{\tau-\xi}} d\xi - \\ & - \frac{R_1}{\lambda} \frac{1}{\pi^2 i} \int_0^{\sigma+i\infty} \int_{\sigma-i\infty}^{\sigma+i\infty} \exp(\sigma\tau) \frac{I_1\left(R_1\sqrt{p^2 + \frac{s}{a}}\right) K_0\left(r\sqrt{p^2 + \frac{s}{a}}\right)}{\sqrt{p^2 + \frac{s}{a}}} \times \end{aligned} \quad (2)$$

$$\times \bar{q}(s) \cos pz ds dp - \frac{R_2}{\lambda} \frac{1}{\pi^2 i} \int_0^\infty \int_{\sigma-i\infty}^{\sigma+i\infty} \exp(s\tau) \times \frac{K_1\left(R_2 \sqrt{p^2 + \frac{s}{a}}\right) I_0\left(r \sqrt{p^2 + \frac{s}{a}}\right)}{\sqrt{p^2 + \frac{s}{a}}} \bar{q}(s) \cos pz ds dp; \quad (3)$$

$$\Delta T_3(r, z, \tau) = \frac{R_2}{\lambda} \frac{1}{\pi^2 i} \int_0^\infty \int_{\sigma-i\infty}^{\sigma+i\infty} \exp(s\tau) \times \frac{I_1\left(R_2 \sqrt{p^2 + \frac{s}{a}}\right) K_0\left(r \sqrt{p^2 + \frac{s}{a}}\right)}{\sqrt{p^2 + \frac{s}{a}}} \bar{q}(s) \cos pz ds dp -$$

$$- \frac{R_1}{\lambda} \frac{1}{\pi^2 i} \int_0^\infty \int_{\sigma-i\infty}^{\sigma+i\infty} \exp(s\tau) \frac{I_1\left(R_1 \sqrt{p^2 + \frac{s}{a}}\right) K_0\left(r \sqrt{p^2 + \frac{s}{a}}\right)}{\sqrt{p^2 + \frac{s}{a}}} \times \bar{q}(s) \cos pz ds dp.$$

Setting  $R_1 = 0$ ,  $R_2 = \infty$ ,  $q(\xi) = q_0 = \text{const}$  in Eq. (3), a one-dimensional solution for a semiinfinite body is obtained for the case when a constant-power heat source is acting at its surface [1]. When  $R_1 = 0$ ,  $R_2 = r_0$ ,  $q(\xi) = q_0 = \text{const}$ ,  $r = 0$  (solution at the axis  $z \geq 0$ ), an expression is obtained for the excess temperature [2, 3]

$$\Delta T_2(0, z, \tau) = \frac{2q_0 \sqrt{\tau}}{b} \left[ \text{ierfc}\left(\frac{z}{2\sqrt{a\tau}}\right) - \text{ierfc}\left(\frac{\sqrt{r_0^2 + z^2}}{2\sqrt{a\tau}}\right) \right]. \quad (5)$$

Setting  $z = 0$  in Eq. (5), a solution is obtained for the central point of a constant-power circular heat source acting at the surface of a semiinfinite body [4, 5].

Consider the particular case of the solution of Eq. (2) when  $r = 0$ , i.e., at the axis  $z \geq 0$

$$\Delta T_1(0, z, \tau) = \frac{R_1}{\lambda} \frac{1}{\pi^2 i} \int_{\sigma-i\infty}^{\sigma+i\infty} \bar{q}(s) \exp(s\tau) I_{s1}(pz) ds - \frac{R_2}{\lambda} \frac{1}{\pi^2 i} \int_{\sigma-i\infty}^{\sigma+i\infty} \bar{q}(s) \exp(s\tau) I_{s2}(pz) ds, \quad (6)$$

where

$$I_{sn}(pz) = \sqrt{\frac{\pi z}{2}} \int_0^\infty J_{-\frac{1}{2}}(pz) \frac{K_1\left(R_n \sqrt{p^2 + \frac{s}{a}}\right)}{\sqrt{p^2 + \frac{s}{a}}} \sqrt{p} dp \quad (n = 1, 2). \quad (7)$$

The integral in Eq. (7) is a particular case of the Sonin-Gegenbauer integral [6]. Calculating  $I_{sn}(pz)$  and integrating Eq. (6) with respect to the variable  $s$ , it is found that

$$\Delta T_1(0, z, \tau) = \frac{1}{b\sqrt{\pi}} \int_0^\tau \frac{q(\xi)}{\sqrt{\tau-\xi}} \left\{ \exp\left[-\frac{R_1^2 + z^2}{4a(\tau-\xi)}\right] - \exp\left[-\frac{R_2^2 + z^2}{4a(\tau-\xi)}\right] \right\} d\xi. \quad (8)$$

When  $z = 0$  (the solution at the surface of a semiinfinite body at a point coinciding with the center of the heater ring)

$$\Delta T_1(0, 0, \tau) = \frac{1}{b\sqrt{\pi}} \int_0^\tau \frac{q(\xi)}{\sqrt{\tau-\xi}} \left\{ \exp\left[-\frac{R_1^2}{4a(\tau-\xi)}\right] - \exp\left[-\frac{R_2^2}{4a(\tau-\xi)}\right] \right\} d\xi. \quad (9)$$

If the specific heat flux  $q(\tau)$  is specified in the form of a rectangular pulse, that is

$$q(\tau) = \begin{cases} q_0 = \text{const}, & 0 < \tau < \tau_0; \\ 0, & \tau > \tau_0, \end{cases} \quad (10)$$

then

$$\bar{q}(s) = \int_0^\infty q(\tau) \exp(-s\tau) d\tau = q_0 \left\{ \frac{1}{s} [1 - \exp(-\tau_0 s)] \right\}; \quad (11)$$

$$L^{-1} \left[ \frac{q_0}{s} \{1 - \exp(-\tau_0 s)\} \right] = q(\tau) = q_0 [1 + U(\tau_0 - \tau) - U(\tau_0)],$$

where  $U(\tau_0 - \tau)$  and  $U(\tau_0)$  are unit symmetric functions [7]. Substituting the value  $q(\xi) = q_0 [1 + U(\tau_0 - \xi) - U(\tau_0)]$  into Eq. (9) and integrating, an expression is obtained for the change in excess temperature  $\Delta T_1(0, 0, \tau)$  at the surface of a semiinfinite body at a point coinciding with the center of the annular heater under the pulsed action of a heat flux

$$\begin{aligned} \Delta T_1(0, 0, \tau) = & \frac{2q_0 \sqrt{\tau}}{b} \left[ \operatorname{ierfc} \left( \frac{R_1}{2\sqrt{a\tau}} \right) - \operatorname{ierfc} \left( \frac{R_2}{2\sqrt{a\tau}} \right) \right] - \\ & - U(\tau - \tau_0) \frac{2q_0 \sqrt{\tau - \tau_0}}{b} \left[ \operatorname{ierfc} \left( \frac{R_1}{2\sqrt{a(\tau - \tau_0)}} \right) - \operatorname{ierfc} \left( \frac{R_2}{2\sqrt{a(\tau - \tau_0)}} \right) \right]. \end{aligned} \quad (12)$$

Equation (12) may be rewritten in the form

$$\begin{aligned} \Theta^*(F_0, \varphi_0, K_R) = & \frac{\Theta(0, 0, \tau)}{K_i} = 2\sqrt{F_0} \left\{ \left[ \operatorname{ierfc} \left( \frac{1}{2\sqrt{F_0}} \right) - \right. \right. \\ & \left. \left. - \operatorname{ierfc} \left( \frac{K_R}{2\sqrt{F_0}} \right) \right] - U(F_0 - F_{00}) \sqrt{1 - \varphi_0} \left[ \operatorname{ierfc} \left( \frac{1}{2\sqrt{(1 - \varphi_0)F_0}} \right) - \right. \right. \\ & \left. \left. - \operatorname{ierfc} \left( \frac{K_R}{2\sqrt{(1 - \varphi_0)F_0}} \right) \right] \right\}. \end{aligned} \quad (13)$$

Curves of  $\Theta^*(F_0, \varphi_0, K_R)$  at  $F_{00} = 0.04$  and various values of  $K_R$  are shown in Fig. 1.

Determining the thermophysical characteristics of the given semiinfinite body by a pulsed method entails finding the time of onset of the excess-temperature maximum ( $\tau = \tau_{\max}$ ) at the center of an annular heater at the body surface and  $\Delta T_{1\max}$  at this time. Note that it follows from the physical representation of the problem that the excess-temperature maximum at the given point sets in for any bodies with different thermophysical properties at  $\tau > \tau_0$ . In this case,  $U(F_0 - F_{00})$  in Eq. (13) is equal to unity.

To find the maximum of the function  $\Theta^*(F_0, \varphi_0, K_R)$ , the first derivative of this function with respect to  $F_0$  must be set equal to zero

$$\begin{aligned} & \exp \left[ -\frac{1}{4F_{0\max}} \right] - \exp \left[ -\frac{K_R^2}{4F_{0\max}} \right] - \sqrt{1 - \varphi_0^*} \times \\ & \times \left\{ \exp \left[ -\frac{1}{4(1 - \varphi_0^*)F_{0\max}} \right] - \exp \left[ -\frac{K_R^2}{4(1 - \varphi_0^*)F_{0\max}} \right] \right\} = 0. \end{aligned} \quad (14)$$

Since Eq. (14) is analytically unsolvable relative to  $F_{0\max}$ , its nonzero roots are determined by the numerical method of Newton [8]. The values of the roots, calculated with an accuracy of  $10^{-7}$  are various  $\varphi_0^*$ , are shown in Table 1 ( $K_R = 1.25$ ).

Thus, specifying the specific heat flux  $q_0$  and its duration of action  $\tau_0$  in the experiment, the excess-temperature maximum at the center of the annular heater at the surface of the given thermally semiinfinite body  $\Delta T_1(0, 0, \tau_{\max})$  and the time of onset of this maximum  $\tau_{\max}$  are measured. Calculating  $\varphi_0^* = \tau_0/\tau_{\max}$ , the corresponding value of  $F_{0\max}$  is found from the resulting value of  $\varphi_0^*$  and Table 1, and hence the thermal diffusivity of the body is determined

$$a = \frac{F_{0\max} R_1^2}{\tau_{\max}}. \quad (15)$$

The other thermophysical characteristics are calculated on the basis of a modified form of Eq. (12)

$$\Delta T_1(0, 0, \tau_{\max}) = \frac{2q_0 \sqrt{\tau_{\max}}}{b} \varphi_b(F_0, \varphi_0, K_R), \quad (16)$$

TABLE 1. Values of the Roots  $Fo_{\max}$  of Eq. (14) and the Corresponding Values of  $\varphi_b(Fo, \varphi_0, K_R)$

$\varphi_0^*$	$Fo_{\max}$	$\varphi_b$	$\varphi_0^*$	$Fo_{\max}$	$\varphi_b$
0,01	0,20935743	0,00041806	0,51	0,30361471	0,02488005
0,03	0,21151167	0,00126056	0,53	0,31075311	0,02605740
0,05	0,21374092	0,00211176	0,55	0,31842643	0,02725600
0,07	0,21604953	0,00297192	0,57	0,32670210	0,02847676
0,09	0,21844216	0,00384128	0,59	0,33565977	0,02972057
0,11	0,22092393	0,00472015	0,61	0,34539431	0,03098835
0,13	0,22350029	0,00560880	0,63	0,35601968	0,03228102
0,15	0,22617724	0,00650754	0,65	0,36767413	0,03359942
0,17	0,22896130	0,00741671	0,67	0,38052694	0,03494432
0,19	0,23185961	0,00833663	0,69	0,39478833	0,03631632
0,21	0,23487990	0,00926766	0,71	0,41072231	0,03771578
0,23	0,23803073	0,01021018	0,73	0,42866586	0,03914261
0,25	0,24132145	0,01116459	0,75	0,44905646	0,04059607
0,27	0,24476245	0,01213130	0,77	0,47247321	0,04207440
0,29	0,24836504	0,01311076	0,79	0,49970092	0,04357422
0,31	0,25214189	0,01410343	0,81	0,53183214	0,04508957
0,33	0,25610707	0,01510981	0,83	0,57043821	0,04661032
0,35	0,26027612	0,01613041	0,85	0,61787112	0,04811941
0,37	0,26466655	0,01716578	0,87	0,67782916	0,04958775
0,39	0,26929802	0,01821651	0,89	0,75651092	0,05096443
0,41	0,27419247	0,01928320	0,91	0,86522813	0,05215596
0,43	0,27937494	0,02036650	0,93	1,02725400	0,05297756
0,45	0,28487365	0,02146710	0,95	1,30013182	0,05301752
0,47	0,29072096	0,02258572	0,97	1,88093677	0,05114154
0,49	0,29695380	0,02372310	0,99	4,33630832	0,04215940

where

$$\varphi_b(Fo, \varphi_0, K_R) = \operatorname{ierfc}\left(\frac{1}{2\sqrt{Fo_{\max}}}\right) - \operatorname{ierfc}\left(\frac{K_R}{2\sqrt{Fo_{\max}}}\right) - \sqrt{1-\varphi_0^*} \times \left[ \operatorname{ierfc}\left(\frac{1}{2\sqrt{(1-\varphi_0^*)Fo_{\max}}}\right) - \operatorname{ierfc}\left(\frac{K_R}{2\sqrt{(1-\varphi_0^*)Fo_{\max}}}\right) \right].$$

Values of  $\varphi_b$  as a function of  $Fo_{\max}$  and  $\varphi_0^*$  for  $K_R = 1.25$  are shown in Table 1.

The thermal-activity coefficient  $b$  is calculated from Eq. (16)

$$b = \frac{2q_0 \sqrt{\tau_{\max}}}{\Delta T_1(0, 0, \tau_{\max})} \varphi_b(Fo, \varphi_0, K_R). \quad (17)$$

The thermal conductivity  $\lambda$  and specific heat  $c_p$  are determined from the relations between the thermophysical characteristics

$$\lambda = b \sqrt{a}; \quad c_p = \frac{\lambda}{a}. \quad (18)$$

Now consider the case when  $R_2 \gg R_1$ , i.e.,  $K_R \rightarrow \infty$ . In Eq. (12) in this case

$$\operatorname{ierfc}\left(\frac{R_1}{2\sqrt{a\tau}}\right) \gg \operatorname{ierfc}\left(\frac{R_2}{2\sqrt{a\tau}}\right), \quad (19)$$

$$\operatorname{ierfc}\left(\frac{R_1}{2\sqrt{a(\tau-\tau_0)}}\right) \gg \operatorname{ierfc}\left(\frac{R_2}{2\sqrt{a(\tau-\tau_0)}}\right). \quad (20)$$

Then the terms containing  $R_2$  in Eq. (12) may be neglected, with a certain error

$$\Delta T_1(0, 0, \tau) = \frac{2q_0}{b} \left\{ \sqrt{\tau} \operatorname{ierfc}\left(\frac{R_1}{2\sqrt{a\tau}}\right) - U(\tau - \tau_0) \operatorname{ierfc}\left(\frac{R_1}{2\sqrt{a(\tau-\tau_0)}}\right) \right\}. \quad (21)$$

It is simple to reduce Eq. (21) to the form

$$\Theta_i^*(Fo, \varphi_0) = \frac{\Theta(0, 0, \tau)}{Ki} = 2\sqrt{Fo} \left[ \operatorname{ierfc}\left(\frac{1}{2\sqrt{Fo}}\right) - U(Fo - Fo_0) \times \sqrt{1-\varphi_0} \operatorname{ierfc}\left(\frac{1}{2\sqrt{(1-\varphi_0)Fo}}\right) \right]. \quad (22)$$

A curve of  $\Theta_1^*(Fo, \varphi_0)$  ( $K_R = \infty$ ) is shown in Fig. 1.

The relative error  $\delta$  arising in Eq. (22) as  $R_2 \rightarrow \infty$  with respect to Eq. (13) is given by the formula

$$\delta = \left| \frac{\Theta^*(Fo, \varphi_0, K_R) - \Theta_1^*(Fo, \varphi_0)}{\Theta^*(Fo, \varphi_0, K_R)} \right| \cdot 100\%. \quad (23)$$

Curves of  $\delta$  as a function of  $K_R = R_2/R_1$  when  $Fo_0 = 0.04$  and at various  $Fo$  are shown in Fig. 2.

It is evident from an analysis of these curves that the relative error is no greater than 0.008% when  $Fo \leq 0.65$  and  $K_R = 5$ . Thus, for  $Fo \leq 0.65$ , the radius  $R_2 = 5R_1$  may be regarded as infinite with a relative error for  $\Theta_1^*(Fo, \varphi_0)$  of 0.008%.

Analyzing the function  $\Theta_1^*(Fo, \varphi_0)$  in Eq. (22) at the maximum when  $\tau > \tau_0$ , the condition for the maximum will be determined by the following equation

$$[\Theta_1^*(Fo, \varphi_0)]'_{Fo} = \frac{1}{\sqrt{\pi} \sqrt{Fo_{\max}}} \left[ \exp\left(-\frac{1}{4Fo_{\max}}\right) - \frac{1}{\sqrt{1-\varphi_0^*}} \exp\left(-\frac{1}{4(1-\varphi_0^*)Fo_{\max}}\right) \right] = 0 \quad (24)$$

or

$$\exp\left(-\frac{1}{4Fo_{\max}}\right) - \frac{1}{\sqrt{1-\varphi_0^*}} \exp\left(-\frac{1}{4(1-\varphi_0^*)Fo_{\max}}\right) = 0, \quad (25)$$

where  $\varphi_0^* = Fo_0/Fo_{\max} = \tau_0/\tau_{\max}$ .

Taking logarithms in Eq. (25) gives

$$\frac{\varphi_0^*}{4Fo_{\max}(1-\varphi_0^*)} = \ln \sqrt{\frac{1}{1-\varphi_0^*}},$$

and hence

$$Fo_{\max} = \frac{\varphi_0^*}{4(1-\varphi_0^*) \ln \sqrt{\frac{1}{1-\varphi_0^*}}}. \quad (26)$$

Thus, specifying the specific heat flux  $q_0$  and its duration of action  $\tau_0$  in the experiment and measuring the maximum of the excess temperature at the surface of the given body at the point coinciding with the center of the annular heater  $\Delta T_1(0, 0, \tau_{\max})$  and also the time of onset of this maximum  $\tau_{\max}$ , the value  $\varphi_0^* = \tau_0/\tau_{\max}$  may be calculated. Then  $Fo_{\max}$  is determined from Eq. (26). The thermal diffusivity is calculated from Eq. (15) and the thermal-activity coefficient from Eq. (17), taking into account, however, that in this case

$$\varphi_b(Fo, \varphi_0, K_R) = \varphi_b(Fo, \varphi_0) = \text{ierfc}\left(\frac{1}{2\sqrt{Fo_{\max}}}\right) - \sqrt{1-\varphi_0^*} \text{ierfc}\left(\frac{1}{2\sqrt{(1-\varphi_0^*)Fo_{\max}}}\right).$$

The thermal conductivity and the volume specific heat are calculated from Eq. (18).

#### NOTATION

$T_i(r, z, \tau)$ ,  $\Delta T_i(r, z, \tau) = T_i(r, z, \tau) - T_0$ , temperature fields and excess temperatures of a semiinfinite body in the corresponding ranges of  $r$  (see text);  $R_2, R_1, r$ , external and internal radii of annular heat source and current radius;  $q_0$ , heat-flux density;  $\tau_0$ , time of action of rectangular heat-flux pulse;  $a, \lambda, b, c_p$ , thermal diffusivity, thermal conductivity thermal activity, and volume specific heat of semiinfinite body;  $z, \tau$ , current coordinate normal to surface of semiinfinite body and current time;  $Ki = q_0 R_1 / \lambda T_0$ , Kirpichev criterion;  $Fo = a\tau/R_1^2$ ,  $Fo_0 = a\tau_0/R_1^2$ , Fourier number;  $K_R = R_2/R_1$ , ratio of external and internal radii of annular heat source;  $\varphi_0 = \tau_0/\tau = Fo_0/Fo$ , relative time;  $\Theta(0, 0, \tau) = \Delta T_1(0, 0, \tau)/T_0$ , dimensionless relative temperature at surface of semiinfinite body at center of annular heat source;  $U(\tau - \tau_0) = U(Fo - Fo_0)$ , unit symmetric Heaviside function;  $\tau_{\max}$ , time of onset of excess-temperature maximum;  $Fo_{\max} = a\tau_{\max}/R_1^2$ , Fourier number at time  $\tau = \tau_{\max}$ ;  $\Delta T_{1\max} = \Delta T_1(0, 0, \tau_{\max})$ , maximum excess temperature;  $\text{ierfc} X = \int_x^\infty \text{erfc} t dt$  multiple probability integral;  $p, s$ , parameters of infinite integral Fourier cosine transformation and Laplace transformation.

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EFFECTIVE THERMAL CONDUCTIVITY OF ALUMINUM OXIDE WITH METALLIC  
FILLERS IN GASEOUS MEDIA AND A VACUUM AT VARIOUS TEMPERATURES

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Experimental data on  $\lambda$  of granular porous aluminum oxide as a function of copper concentration and temperature in various gaseous media at atmospheric pressure and in a vacuum ( $P = 8 \cdot 10^{-3}$  mm Hg) are presented.

Granular porous aluminum oxide is used in the production of ceramics, refractories, forms, catalysts, etc. Despite its wide use, the literature offers practically no data on its thermophysical properties. At present only the thermophysical properties of monolithic aluminum oxide have been studied sufficiently thoroughly [1].

The effective thermal conductivity coefficient of the aluminum oxide most widely used in high-temperature catalytic processes was studied (specific surface,  $123 \text{ m}^2/\text{g}$ ; total pore volume,  $0.35 \text{ cm}^3/\text{g}$ ; bulk density,  $1 \text{ g}/\text{cm}^3$ ; cylindrical granule dimensions,  $0.8\text{--}1.25 \text{ mm}$ ). The copper-containing specimens were prepared by steeping the aluminum oxide in a solution of copper in nitric acid with subsequent thermal processing in air and hydrogen at  $673^\circ\text{K}$ .

Effective thermal conductivity was measured by a regular thermal regime cylindrical bicalorimeter [2]. The experimental arrangement consisted of the cylindrical bicalorimeter, temperature stabilization system, vacuum system, and filling system. The calorimeter consisted of two coaxially arranged copper cylinders: internal (diameter  $15.95 \text{ mm}$ ) and external (diameter of inner and outer sections  $28.28 \text{ mm}$  and  $90.0 \text{ mm}$ ). The free space between the cylinders was filled with the specimen under study. The specimen thickness was  $6.165 \text{ mm}$ , and the temperature head across its boundaries with  $1.78\text{--}0.90^\circ\text{K}$ . During experiments the bicalorimeter temperature was maintained constant to an accuracy of  $0.005\text{--}0.02^\circ\text{K}$ . Relative measurement uncertainty at a confidence level of  $\alpha = 0.95$  was  $3.2\%$ .

The specimen effective thermal conductivity was determined for the freely poured state. According to Table 1, the effective thermal conductivity increases with increase in copper content.

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